Finite width effects and gauge cancellations in W- and Z-boson production in framework of modified perturbation theory

M. L. Nekrasov

Institute for High Energy Physics, 142284 Protvino, Russia

Abstract

The processes of unstable W- and Z-boson production are considered in a recently proposed modified perturbation theory (PT), based on direct expansion of probabilities instead of amplitudes. In such an approach the nonintegrable singularities in the phase space, which are intrinsic in the conventional PT, appear as singularities in the coupling constant (with subsequent compensation by the decay factors of unstable states). In present paper the systematic research of the modified PT is carried out. The results are compared with the results of the usual approach, based on calculation of the amplitude with Dyson resummation. The solution of the problem of reducing one-loop PT order in the resonance region, leading to paradoxical results in the usual approach, is discovered. On the basis of this solution the proof of gauge cancellations in any order of the modified PT is given. It is shown that within the given accuracy limits the results obtained in the modified PT may be reproduced in the usual approach with addition of an anomalous term to the probability. An elementary generalization of the fermion-loop scheme is established which provides the complete description of W-pair production in the next-to-leading approximation.

1 Introduction

In many field-theoretic applications of the Standard Model, connected with the present and future collider experiments, the effects of instability of W- and Z-bosons should be taken into consideration (as well as of Higgs boson, top quark etc.) [1]. In quantum field theory the usual way to take into account instability consists in Dyson resummation of self-energy of unstable particles [2]. This procedure provides to avoid nonintegrable phase-space singularities caused by contributions of "outgoing" unstable particles. However it makes impossible fixed-order calculations in the framework of perturbation theory (PT). In gauge theories it results [3, 4] in loss of gauge invariance and violation of Ward identities (WI). The latter effect owing to loss of the control of high-energy behavior of the theory can lead to large errors in description of particular processes.

In the case of single Z-boson production (LEP1) the problem of gauge cancellations may be solved *ad hoc* within the precision determined by one-loop corrections to the vertex functions (in fact, this is the next-to-leading-order approximation, NLO). More consistent scheme was proposed in Ref.[5]. In this way only the gauge-invariant contributions to self-energy were Dyson resummed while the gauge-dependent contributions were considered by conventional PT. As a result, the amplitude managed to be presented as a product of gauge-independent factors, two vertex and one resonant. Nevertheless, this result is not universal. Anyway, now it is not clear whether the indicated property of the amplitude will be maintained within the next order of precision determined by two-loop corrections to the vertex functions.

In the case of pair production of unstable particles (LEP2) the amplitude fails to be presented in completely gauge invariant form.[1] The hopes, nevertheless, for any further progress in such calculations are usually connected with rather general idea to determine the minimal set of Feynman diagrams that are necessary for compensating the gauge violation by the Dyson resummed self-energies. With one-loop self-energies are taken into account, in this way the fermion-loop scheme was attracted [8, 9], and also its generalization [10] defined in formalism of the background-field method. The latter scheme allows one to consider of bosonic corrections and remains in force not only in the one-loop approximation. Nevertheless, the desired precision of the description is still unachieved due to the reduction of one-loop PT order in the resonance region [11, 12].

Let us consider in more detail the latter phenomenon. The matter is that the denominator of "unstable" propagator in the resonance region, $p^2 - M^2 = O(g^2)$, is of order $O(g^2)$, but not $O(1)$. So, the one-loop correction to self-energy actually appears in the leading-order approximation, but not in NLO one. Therefore, also the two-loop correction to self-energy is necessary to complete the NLO approximation. However that fact hardly leaves a chance to maintain WI without taking into consideration the two-loop corrections to the vertex functions, which is impractical [11, 12].

In fact, this phenomenon leads to paradoxical consequences. Most clearly it can be seen in the approach of the background-field formalism, which allows one to keep on the validity

[1] It should be noted that besides the considered here approach, based on Dyson resummation, there is an alternate approach called the pole scheme [6]. The gauge invariance in this scheme is initially included, but, unfortunately, an algorithm of evaluation of the corrections is not developed. Anyway, the actual calculations in this scheme do not proceed beyond the double-pole approximation [7]. However the range of applicability of that approximation is rather restricted, and its accuracy is rather low.

of WI provided that with Dyson resummed self-energies up to n-loops the corrections to the vertex functions up to n-loops are taken into account, as well [10]. Let us assume that we know the amplitude calculated in this way up to $(n+1)$-loop corrections. Then, due to the reduction of one-loop PT order in the resonance region, this amplitude is actually known within the n-loop precision (i.e. within g^{2n}-precision). Recall, that the $(n+1)$-loop correction to self-energy in the denominator of unstable propagator is necessary to provide this precision. But, at the same time, the $(n+1)$-loop corrections to the vertex functions are superfluous since they contribute in the next order of expansion in the coupling. Therefore they cannot be relevant in maintaining WI and ensuring the gauge cancellations in the amplitude determined within the n-loop precision. Nevertheless, the WI are remained to be valid due to explicitly these corrections, because the corrections to both the self-energies and vertices are needed to maintain WI [10]. From the usual understanding there is not explanation to simultaneous existence of both these effects.

The solution to this paradox may be given in the framework of the modified PT [13]. Its basic idea is to expand in powers of the coupling directly probabilities instead of amplitudes (the amplitudes prior to calculation of probabilities are considered to be not expanded). Such an order of operations allows one to trace the fundamental connection between the origin of the nonintegrable singularity of the kind $|p^2 - M^2|^{-2}$ in the phase space and the reduction of one-loop PT order in the resonance region. In order to show this property let us remind that the probability is always determined as an integral over kinematic variables of the squared renormalized amplitude (with some weight), and until the integrand is not expanded the integral is convergent due to the imaginary part of the Dyson resummed self-energy that regularizes the kinematic divergence. The fact that the integral becomes divergent after the PT expansion of the integrand (i.e. at $g^2 \to 0$) means that the integral as a function of the coupling constant includes a singularity in g^2. So, in the modified PT approach the kinematic singularities transform into singularities in the coupling constant. As a matter of fact this is the very effect of the mentioned above reduction in the resonance region.

Note, that proceeding only from an amplitude it is impossible to estimate in a mathematically correct way the remainder of the expansion of probability, since in the resonance region the expansion of the amplitude faces an uncertainty $0/0$. (In more radical way this statement is sometimes recognized as an inapplicability of the usual PT in the presence of unstable fundamental particles.) However, proceeding directly from probability, which is an integral, one can do such an estimate. Moreover, within the given precision one can reproduce in probability the contribution of the $(n+1)$-loop correction to self-energy in the denominator of unstable propagator in form of an additive anomalous term. Owing to its additivity it becomes possible in probability to give an independent proof of gauge cancellations in the contributions that are generated by this anomalous term. Let us emphasize that this result does not mean that the inclusion of $(n+1)$-loop corrections to only the denominator of unstable propagator does not lead to violation of WI within the n-loop precision. It means only that the contributions, that violate WI, turn out to be beyond the given precision in probability.

The present paper elaborates in detail the above propositions and on this basis presents the proof of gauge cancellation in any order of the modified PT expansion. Notice, that the latter outcome was practically anticipated in the pioneering work [13], offered the modified PT. Also this work showed that owing to application of the asymptotic operation (AO)

[14, 15, 16] the probability can be presented as the complete expansion in powers of the coupling constant, irrespective of the presence of the weight function in the integral for probability. However the reasoning of Ref.[13], in the part that concerns the problem of gauge cancellations, was not complete since being based on a comparison with results of the usual approach it overlooked the problem of reduction of one-loop PT order in the resonance region and omitted the problem of remainder of the expansion of amplitude. The present paper fills up this gap.

Another aspect of the present paper is the systematic research of the modified PT method. In particular, the independence of the formalism from the ultraviolet (UV) renormalization scheme has been shown and the recurrent relation for the infrared (IR) counterterms — the specific ingredients of the AO formalism — has been deduced. The most important outcome, however, which is more involved in practice, is the construction of an elementary generalization of the fermion-loop scheme, which allows one to describe processes with unstable vector bosons production ensuring both the gauge cancellations and NLO precision in the sense of expansion in powers of the coupling constant.

This paper is organized as follows. In Section 2 a general statement of the problem of AO expansion is expounded. In Section 3 the basic formulas of the modified PT are derived (on the whole, the content of this section follows Ref.[13]). The properties of the AO expansion of squared unstable propagator are studied in Section 4. In particular, the basic formula (23) is derived, which helps us to solve the above-mentioned paradox (in Section 6). Section 5 discusses the soft-photon problem. In Section 6 the general proof of gauge cancellations in processes mediated by unstable particles is given. Section 7 is devoted to construction of an elementary generalization of the fermion-loop scheme. In Section 8 results are discussed.

2 Unstable propagator in AO, the statement of the problem

The problems concerned in this section are connected exclusively to the structure of denominator of the propagator of unstable particle (independently from its type). So, rejecting (temporarily) all factors in the numerator, let us represent the propagator in the form

$$\Delta(\alpha;\tau) = \frac{1}{M^2 - p^2 - \Sigma} \;=\; \frac{1}{\tau - \alpha h(\tau) - \mathrm{i}\,\alpha f(\tau)}\,. \tag{1}$$

Here $\alpha = g^2/(2\pi)$ is the squared coupling constant, $\tau = M^2 - p^2$ is a kinematic variable, M and p are the mass and 4-momentum of unstable particle (here and after M^2 is considered without the usual imaginary addend $-\mathrm{i}\varepsilon$), Σ stands for the renormalized self-energy,[2] αh and αf are its real and imaginary parts with the extracted for convenience factor α. Here we do not declare the scheme of UV renormalization in which the propagator (1) is determined since the results will not depend on it (see below).[3] The property of instability by definition means

[2]In the case of vector bosons the self-energy includes two Lorentz-covariant structures, which are proportional to $g_{\mu\nu}$ and $p_\mu p_\nu$. In propagator (1) the contribution only of the first structure is taken into account. The second structure contributes to nonphysical pole term, which will be analyzed separately in Section 6.

[3]An important point is that the variation of the mass M^2, accompanied by transition from one scheme of UV renormalization to another, is of order $O(\alpha)$.

that $f \neq 0$ in some neighborhood of a point $\tau = 0$. Owing to causality we assume $f > 0$ in this neighborhood. In what follows we suppose that the size of this neighborhood amounts the magnitude of order $O(\alpha)$ and that it involves a solution to the equation $\tau - \alpha h(\tau) = 0$. Function $h(\tau)$, generally speaking, may be nonzero at $\tau = 0$.

The probability of a process of production of unstable particle is defined by an integral over some kinematic region of the squared propagator $\mathcal{W}(\alpha;\tau)$ with some weight function,

$$P(\alpha) = \int d\tau\; \varphi(\tau)\; \mathcal{W}(\alpha;\tau), \tag{2}$$

$$\mathcal{W}(\alpha;\tau) \equiv \left|\Delta(\tau)\right|^2 = \frac{1}{\left[\tau - \alpha\, h(\tau)\right]^2 + \alpha^2 f^2(\tau)}. \tag{3}$$

The weight function $\varphi(\tau)$ corresponds, first of all, to the complementary part of the diagram of unitarity, describing probability, with respect to the given squared propagator. In processes with charged initial and final states it includes the effects of the photon radiation from initial and final states (convolution). Moreover, the weight function includes the hardware factors (aperture, etc.) of the experimental devices.

Function $\mathcal{W}(\alpha;\tau)$ in formula (3) by virtue of the property $f \neq 0$ is finite and, therefore, integrable in the neighborhood of $\tau = 0$. However in the limit $\alpha \to 0$ in $\mathcal{W}(\alpha;\tau)$ there appears a nonintegrable singularity $1/\tau^2$. Usually this fact is interpreted as an indication on impossibility of direct application of PT and expansion in the coupling constant in the presence of unstable particles [1]-[12]. Nevertheless, the expansion exists for probability $P(\alpha)$. Really, the origin of nonintegrable singularity from only the mathematical point of view means that the *result of integration* of $\mathcal{W}(\alpha;\tau)$ with weight function $\varphi(\tau)$ involves a singularity in α at $\alpha \to 0$. If this singularity would manage to be extracted and turn out a power one, then the expansion of the integral in series of the coupling would be possible, with the only difference that it will be expansion of Laurent instead of Taylor. (Let us notice, that the weight function $\varphi(\tau)$ actually is a power-dependent one in parameter α. Therefore, the expansion of the whole integral may ultimately take form of a Taylor expansion. However *a priori* this property is not obvious. So, to study the problem we first consider $\varphi(\tau)$ as an arbitrary rather smooth test function, which is generally nonzero at $\tau = 0$ and *does not depend on parameter* α.)

To study the type of the mentioned above singularity in α let us use the property that the singularity results from integration over the small τ, and keep in functions $h(\tau)$ and $f(\tau)$ only their leading terms in asymptotic expansion at $\tau \to 0$. In other words, let us approximate them by $h_0 = h(0)$ and $f_0 = f(0)$. As a result, up to corrections which are inessential for the leading contribution (the corrections will be calculated), we obtain the following approximation for $\mathcal{W}(\alpha;\tau)$:

$$\mathcal{W}(\alpha;\tau) \cong \frac{1}{[\tau - \alpha\, h_0]^2 + \alpha^2 f_0^2}\,. \tag{4}$$

From here it follows by virtue of homogeneity that the integration over τ leads to singularity $1/\alpha$. Indeed, let us divide the range of integration on $|\tau| < \mathrm{const} \times \alpha$ and $|\tau| > \mathrm{const} \times \alpha$ with enough large const. Then, the integration over the second range gives finite contribution while on the first one it gives the mentioned above singularity. This can be easily shown by changing the variable of integration and taking into consideration the property $\varphi(0) \neq 0$.

Moreover, we can claim that the coefficient at the singularity is proportional to f_0^{-1} and does not depend on h_0. Really, f_0 may be included into a normalization of α, whereas h_0 does not make contribution to the leading term of the expansion, since while setting $h_0 = 0$ the function $\mathcal{W}(\alpha;\tau)$ does not face singularity in τ. On similar reasons the weight function $\varphi(\tau)$ gives contribution to the leading term as a trivial factor $\varphi(0)$.

So, in spite of the fact that the expansion of $\mathcal{W}(\alpha;\tau)$ in α under the sign of the integral is an incorrect operation, the expansion of the *result of integration* is sensible. Moreover, some properties of this expansion can be determined before actual calculation of the integral. For systematic such calculations there is a special method called asymptotic operation, AO [14, 15, 16]. The key point in AO is transition to the extended interpretation of an integrand as a product of the kernel of a *generalized* function on a *test* function [17, 18]. (In fact this means interpretation of the integral as a continuous linear functional on test functions.) When the integral is well defined (the integral *before* expansion of an integrand) the mentioned above generalization means no changes. However after the formal expansion of the integrand the new interpretation allows one to make sense to the nonintegrable terms of the integrand.

Thus, the problem of expansion of the integrand basically may be solved through the method of generalized functions. Then, it is necessary to see to asymptotic properties of the expansion. In the AO framework for this purpose the property of ambiguity of the extension of nonintegrable functions is used. Generally it is well-known, e.g. from experience of UV renormalizations, that elimination of divergences is usually accompanied by appearance of ambiguities. When an integral is determined by the method of generalized functions, the ambiguities are described through addition to a generalized function of the so-called counterterms which are proportional to the delta-function or its derivatives, located strictly in the point of nonintegrable singularity.[4] In the AO framework the values of the coefficients at counterterms are unambiguously fixed by requirement of reconstruction of that (exact) result which may be obtained directly from the expansion of the initial integral. (Let us remind, that before expansion of the integrand the integral was well defined and there were no ambiguities in it.) Moreover, AO presents a practical recipe of calculation of these coefficients in each order of the expansion prior to calculation of the integral. The resulting counterterms contain complete information about the singular terms in the expansion parameter. Simultaneously the counterterms may contain also nonsingular contributions which correct the asymptotic property of the expansion.

In the above example the counterterm, which describes the leading term of asymptotic expansion of $\mathcal{W}(\alpha;\tau)$, is of the form $c/(\alpha f_0) \times \delta(\tau)$ with c is some numerical factor. In the given case the value of factor c, as well as the very appearance of this counterterm, follows from the well-known formula in the theory of generalized functions, $\lim\limits_{\alpha\to 0} \alpha/(\tau^2+\alpha^2) = \pi\,\delta(\tau)$. With allowance for this formula the expansion of $\mathcal{W}(\alpha;\tau)$ up to $O(1)$ corrections is defined unambiguously and comes to the delta-function only. Up to $O(\alpha)$ corrections the expansion is nontrivial. In the most general form it can be written as

$$\mathcal{W}(\alpha;\tau) = \pi/(\alpha f_0)\,\delta(\tau) + \left[1\big/\tau^2\right] + c_0\,\delta(\tau) + c_1(-)\,\delta'(\tau) + O(\alpha). \tag{5}$$

[4] Let us emphasize, that introduction of counterterms is a general place in the theory of generalized functions (see e.g. [18]). Actually this idea was used by N.N.Bogoliubov [19, 20] for substantiation of the R-operation. In the AO context the term "counterterms" was introduced [14, 15] in order to emphasize an analogy with the theory of UV renormalizations.

Here $[1/\tau^2]$ is the generalized function $1/\tau^2$ defined with some prescription. (The square brackets mean the presence of the prescription. The most common, but not mandatory prescription is the principal value). The next terms in (5) with the delta-function and its derivative describe the counterterms which correct the contribution of $[1/\tau^2]$.

In general case the complete definition of the generalized function $1/\tau^2$ may be done like as follows [18]. First, one may do two subtractions in the test function $\varphi(\tau)$ in some neighborhood of $\tau = 0$ by replacing $\varphi(\tau)$ to expression $\varphi(\tau) - \varphi(0) - \tau\varphi'(0)$. As a result, the nonintegrable singularity $1/\tau^2$ becomes compensated. (In fact this is one of possible prescriptions.) Then, in order to describe ambiguity emerging with this subtraction, one should add two counterterms to $1/\tau^2$, one proportional to delta-function and another to its first derivative (both counterterms correspond to the subtractions). The coefficients at counterterms must be determined in such a way as to guarantee the asymptotic properties of the expansion in order $O(1)$. Their values depend on h and f, and on the choice of the prescription in $1/\tau^2$, but the sum, taken as a whole, will not depend on the prescription.

The above procedure may be continued. The next term of the formal expansion of $\mathcal{W}(\alpha;\tau)$ is $2\alpha h(\tau) \times 1/\tau^3$. For its complete definition three counterterms are needed which involve the delta-function, it first, and second derivatives. The coefficients at them can be determined by the requirement of keeping on the asymptotic properties of the expansion. The practical recipe of determination of these coefficients is presented in next section.

3 Calculation of counterterms

Let us show the technique of calculation of counterterms on an example of AO expansion of $\mathcal{W}(\alpha;\tau)$ up to $O(\alpha^2)$ corrections. Since the leading term of this expansion is of order $O(\alpha^{-1})$, the mentioned precision is sufficient for construction of the next-to-next-to-leading-order (NNLO) approximation. Although we do not need such precision for the applications considered in this paper, we nevertheless carry out such calculations because it is useful for more complete elucidation of the structure of the AO expansion. As was mentioned above, the general structure of the expansion is as follows:

$$\mathcal{W}(\alpha;\tau) \;=\; \frac{1}{\tau^2} + \frac{2h(\tau)}{\tau^3}\alpha + E(\tau) + O(\alpha^2)\,. \tag{6}$$

Here the first two terms result from the formal expansion of $\mathcal{W}(\alpha;\tau)$ in the sense of usual functions. For definiteness, let us agree to understand the poles in τ in the sense of principal value. Below we remind the most commonly used definitions of the principal value,

$$\mathrm{VP}\frac{1}{\tau^n} = \frac{1}{2}\left[\frac{1}{(\tau+\mathrm{i}0)^n} + \frac{1}{(\tau-\mathrm{i}0)^n}\right] = \frac{(-)^{n-1}}{(n-1)!}\,\frac{\mathrm{d}^n}{\mathrm{d}\tau^n}\ln|\tau|\,. \tag{7}$$

(Both versions are equivalent, see e.g. [18]. The derivatives in the last expression are understood in the sense of generalized functions, i.e. they must be switched to the test function via formal integration by parts without taking into account the boundary terms in the integral.) Quantity $E(\tau)$ in formula (6) represents the sum of counterterms which in this case are proportional to the delta-function, its first, and second derivatives,

$$E(\tau) = \sum_{n=0}^{2} \frac{(-)^n c_n}{n!}\,\delta^{(n)}(\tau). \tag{8}$$

Further in this section we assume that $h(\tau)$ and $f(\tau)$ together with their second derivatives are regular functions in some neighborhood of $\tau = 0$.[5] For simplicity we assume, at first, that functions h and f include one-loop contributions only. The necessary generalizations to the case of multi-loop contributions will be considered in the end of the given section.

Our aim is to determine coefficients c_n, $n = 0, 1, 2$, in such a way as to ensure the following relation with any test function $\varphi(\tau)$ decreasing enough rapidly in the infinity,

$$\int_{-\infty}^{+\infty} d\tau\, \varphi(\tau) \left[\mathcal{W}(\alpha;\tau) - \frac{1}{\tau^2} - \frac{2h(\tau)}{\tau^3}\,\alpha - E(\tau) \right] = O(\alpha^2). \tag{9}$$

Let us note at once, that for the solution to this problem we need not know all information about function $h(\tau)$ in the third term in square brackets, but need know three terms of its asymptotic expansion at small τ,

$$h(\tau) = h_0 + \tau\, h_0' + (\tau^2/2)\, h_0'' + o(\tau^2). \tag{10}$$

This property arises from that fact that the remainder $o(\tau^2)$ cancels the nonintegrable singularity $1/\tau^3$ in formula (9).

Now let us substitute (8) into (9) and, following [14], present the test function $\varphi(\tau)$ as a linear combination of three basis functions $\varphi_n(\tau)$, $n = 0, 1, 2$, satisfying condition $\varphi_n^{(k)}(0) = \delta_n^k$ with δ_n^k is the Kronecker symbol. As the result we obtain

$$c_n = \int_{-\infty}^{+\infty} d\tau\, \varphi_n(\tau) \left[\mathcal{W}(\alpha;\tau) - \frac{1}{\tau^2} - \frac{2h(\tau)}{\tau^3}\,\alpha \right] + O(\alpha^2). \tag{11}$$

From (11) it follows that within the given precision coefficients c_n do not depend on the choice of the test functions. Indeed, in case of other test functions $\widetilde{\varphi}_n(\tau)$ possessing of the same property $\widetilde{\varphi}_n^{(k)}(0) = \delta_n^k$ one obtains coefficients $\widetilde{c}_n$ instead of c_n. However the difference between these coefficients makes up a quantity of order $O(\alpha^2)$, because the difference between the relevant integrals is determined by the weight function $\widetilde{\varphi}_n(\tau) - \varphi_n(\tau)$ which equals zero at $\tau = 0$ together with its first and second derivatives. It is easy to see that the integral in formula (11) with such weight is of order $O(\alpha^2)$. Due to this property, without loss of generality, one may choose the test function $\varphi_n(\tau)$ to be a step-like one. Namely, one may set $\varphi_n(\tau) = \tau^n \times \theta(|\tau| < \Lambda)$. Then,

$$c_n = \int_{-\Lambda}^{+\Lambda} d\tau\, \tau^n \left[\mathcal{W}(\alpha;\tau) - \frac{1}{\tau^2} - \frac{2h(\tau)}{\tau^3}\alpha \right] + O(\alpha^2), \qquad n = 0, 1, 2. \tag{12}$$

Formula (12) basically solves the stated above problem. However it is still too complicated for operational use since through $\mathcal{W}(\alpha;\tau)$ it contains the dependence on generally speaking unknowns functions $h(\tau)$ and $f(\tau)$. Moreover, it contains a lot of superfluous information

[5] If the unstable particle interacts with massless particles (photons) then, strictly speaking, this requirement is not correct. However, by introducing the photon mass one can restore the analyticity inside a neighborhood defined by the generated mass gap. This is enough for our purposes. (See also discussion in Sections 5 and 6.)

because the integral in r.h.s. involves the contributions beyond the given precision. In particular, the dependence on the cutoff parameter Λ is like that.

Both mentioned above problems may be solved through a procedure of homogenization [16]. In the given case it is reduced to the following transformations in the integrand: first we do substitutions $\tau \to \xi\tau$, $\alpha \to \xi\alpha$, then the result expand in powers of ξ, and in the end set $\xi = 1$. Each term of this (secondary) expansion is proved to be a homogeneous function of τ and α. So, it gives strictly definite in the order of α contribution to the integral, which at this stage may be considered without the cutoff. The first term of this expansion gives the leading term $c_n^{(0)}$ of the expansion in α of the coefficient c_n,

$$c_n^{(0)} = \int\limits_{-\infty}^{+\infty} d\tau\, \tau^n \left[\frac{1}{(\tau - \alpha h_0)^2 + \alpha^2 f_0^2} - \frac{1}{\tau^2} - \frac{2h_0}{\tau^3}\alpha\right], \qquad n = 0, 1, 2. \tag{13}$$

The next term of the expansion of homogenization will give the correction term $c_n^{(1)}$, etc. On power count $c_n^{(0)} \sim \alpha^{n-1}$, $c_n^{(1)} \sim \alpha^n$, etc. Adding up the necessary number of $c_n^{(i)}$ one obtains coefficient c_n with required precision.

Let us emphasize, that the integral in formula (13) is convergent in infinity. The similar property of convergence takes place also for other $c_n^{(i)}$ and, moreover, is a general corollary of application of the homogenization [16]. Let us remind, that the singular terms $1/\tau^2$ and $1/\tau^3$ in formula (13) are defined in the sense of principal value (a change of prescription will change correspondingly the values of $c_n^{(i)}$). So, coefficients c_n are well defined throughout.

By carrying out the relevant calculations we come to the following result (for the first time obtained in [13]):

$$\begin{aligned} c_0 &= \frac{\pi}{\alpha f_0} + \frac{\pi\,(h_0' f_0 - h_0 f_0')}{f_0^2} \\ &\quad + \frac{\pi\Big(h_0'^2 f_0^2 + h_0^2 f_0'^2 + h_0 h_0'' f_0^2 - 2h_0 h_0' f_0 f_0' - \frac{1}{2}h_0^2 f_0 f_0'' - \frac{1}{2} f_0^3 f_0''\Big)}{f_0^3}\,\alpha, \\ c_1 &= \frac{\pi h_0}{f_0} + \frac{\pi\,(2h_0 h'_0 f_0 - h_0^2 f'_0 - f_0^2 f'_0)}{f_0^2}\,\alpha\,, \qquad c_2 = \frac{\pi\,(h_0^2 - f_0^2)}{f_0}\,\alpha\,. \end{aligned} \tag{14}$$

Here the subscript 0 means that the relevant quantity is defined at $\tau = 0$, while superscript means derivatives. For example, $h_0' = \mathrm{d}h(\tau)/\mathrm{d}\tau|_{\tau=0}$, etc.

The above result may be written in a more compact form if quantities c_n in formula (8) are treated as functions of τ. In this case one can obtain, instead of (14),

$$c_0 = \frac{\pi}{\alpha f}\,, \qquad c_1 = \frac{\pi h}{f}\,, \qquad c_2 = \frac{\pi\,(h^2 - f^2)}{f}\,\alpha\,. \tag{15}$$

Here c_n, $n = 0, 1, 2$, h and f are understood as the functions of τ. The equivalence of these two forms of notation, (14) and (15), follows from relations $f(\tau)\,\delta'(\tau) = f_0\,\delta'(\tau) - f_0'\,\delta(\tau)$ and $f(\tau)\,\delta''(\tau) = f_0\delta''(\tau) - 2f_0'\delta'(\tau) + f_0''\,\delta(\tau)$.

The above results may be easily extended to the case when functions $h(\tau)$ and $f(\tau)$ involve the multi-loop contributions. In this case in order to obtain the complete AO expansion one should carry out the *usual* expansion in α in formulas (14) or (15), in which h and f are understood as the full functions that involve the multi-loop contributions [13]. The simplest way to prove this property is as follows: assuming that h and f are the full functions one might repeat all the same reasoning as was done above, except that during homogenization one should modify the scaling by setting $\alpha^n \to \xi\alpha^n$ in n-loop contributions. As a result the higher-loop contributions will be identified with the one-loop ones, and formulas (14), (15) will be restored automatically.

4 The properties of the AO expansion of $\mathcal{W}(\alpha;\tau)$

Now let us discuss the properties of the obtained AO expansion. First of all we interesting in those ones which will be useful for substantiation of gauge cancellation in electroweak theory. Notice, by virtue of the specificity of the modified Feynman rules (see formulas (6), (8) and (14)) the solution to this problem *a priori* is not obvious. In this connection we will adhere to the strategy of a comparison of results obtained in the modified PT with those obtained in the usual approach based on calculation of the amplitude. As a tool of comparison we will use some incomplete expansions of $\mathcal{W}(\alpha;\tau)$ which ultimately lead to the same complete AO expansion, but look more conventionally.

Let us begin with explicit demonstration of the property of independence of the result of AO expansion from the sequence of expansion in the higher-loops. In other words, let us explicitly show that the results of the AO expansion of $\mathcal{W}(\alpha;\tau)$ will not vary if instead of (1) one starts with the following formula describing incomplete Dyson resummation:

$$\Delta(\alpha;\tau) = \frac{1}{\tau - \alpha\,\Sigma_1 - \alpha^2\,\Sigma_2 - \alpha^3\,\Sigma_3 + \,\ldots} = \frac{1}{\tau - \alpha\,\Sigma_1} + \frac{\alpha^2\,\Sigma_2 + \alpha^3\,\Sigma_3}{(\tau - \alpha\,\Sigma_1)^2} + \,\ldots \qquad (16)$$

Here $\alpha^n\Sigma_n(\tau)$ stands for the n-loop contribution to self-energy $\Sigma(\alpha;\tau)$. By squaring (16) we obtain an incomplete expansion of $\mathcal{W}(\alpha;\tau)$ each term of which at $\alpha \neq 0$ is an integrable in the usual sense function:

$$\begin{aligned}\mathcal{W}(\alpha;\tau) = \mathcal{W}_1(\alpha;\tau) + \Big[(\alpha^2\,\Sigma_2 + \alpha^3\,\Sigma_3)\;\mathcal{W}_{11}(\alpha;\tau) + \alpha^4\,(\Sigma_2)^2\,\mathcal{W}_{12}(\alpha;\tau) + \text{h.c.}\Big] \\ + \,\alpha^4\,|\Sigma_2|^2\;\mathcal{W}_1^2(\alpha;\tau) + O(\alpha^2)\,.\end{aligned} \qquad (17)$$

Here we have entered new (generalized) functions: $\mathcal{W}_{11}(\alpha;\tau)=\mathcal{W}_1(\alpha;\tau)\Delta_1(\alpha;\tau)$, $\mathcal{W}_{12}(\alpha;\tau)= \mathcal{W}_1(\alpha;\tau)\Delta_1^2(\alpha;\tau)$ and $\mathcal{W}_1^2(\alpha;\tau)=\mathcal{W}_1(\alpha;\tau)\mathcal{W}_1(\alpha;\tau)$, where $\mathcal{W}_1(\alpha;\tau)$ and $\Delta_1(\alpha;\tau)$ are defined as $\mathcal{W}(\alpha;\tau)$ and $\Delta(\alpha;\tau)$ in which Dyson resummed are only the one-loop corrections. Each term in (17) can be completely expanded in the AO sense. By means of the reasoning like that which was done in Section 2 it is easy to show that the leading term of the AO expansion of $\mathcal{W}_{11}(\alpha;\tau)$ has a behavior $1\,/\alpha^2$. So, the first term in square brackets in (17) after an integration with weight function $\varphi(\tau)$ gives contribution of order $O(1)$. The leading terms for $\mathcal{W}_{12}$ and $\mathcal{W}_1^2$ have a behavior $1\,/\alpha^3$. So, the second term in square brackets and the last term in formula (17) are of order $O(\alpha)$. By similar reasoning one can show that the neglected in (17) terms give contributions $O(\alpha^2)$. Thus, with allowance for $\mathcal{W}_1(\alpha;\tau) = O(\alpha^{-1})$, formula (17) describes $\mathcal{W}(\alpha;\tau)$ within the NNLO precision.

It should be especially noted, that the above reasoning is valid as long as $\Sigma_2(\tau)$, $\Sigma_3(\tau)$, etc. with some number of their derivatives are regular functions in some neighborhood of $\tau = 0$.

If it is not so, then their products with the entered above functions $\mathcal{W}^m_{1n}$ must be considered as new generalized functions which properties have to be separately investigated. Such situation takes place when the unstable particle interacts with massless particles. However, when the regularizing mass for massless particles (the soft-mass) is introduced then the problem becomes no longer relevant, because the functions $\Sigma_n(\tau)$ becomes regular in the neighborhood of $\tau = 0$. Actually, the soft-mass singularities are cancelled in probabilities. So, for study qualitative problems the presence of the soft-mass is inessential.

Now let us proceed to the complete (exact) AO expansion of functions $\mathcal{W}^m_{1n}$. For brevity we omit the corresponding derivation since it is similar to that discussed in the previous section. (Let us notice only once again that we mean the case with absence of the massless particles, or with the soft-mass regularization for their contributions.) In accordance with (17) $\mathcal{W}_{11}$ should be expanded up to $O(1)$ corrections, while $\mathcal{W}_{12}$ and $\mathcal{W}^2_1$ up to $O(\alpha^{-2})$ ones. In what follows, however, some next terms will be needed. So let us write down the results beforehand with somewhat exceeding precision:

$$\mathcal{W}_{11}(\alpha;\tau) = E(\tau) + \frac{1}{\tau^3} + O(\alpha), \quad \mathcal{W}_{12}(\alpha;\tau) = E(\tau) + O(1), \quad \mathcal{W}^2_1(\alpha;\tau) = E(\tau) + O(1). \tag{18}$$

Here in all cases the counterterm $E(\tau)$ is still defined by (8), but coefficients c_n in each case are different. In the compact form, in which they are defined as functions on τ, we have:

$$\mathcal{W}_{11}: \qquad c_0 = \frac{\mathrm{i}\pi}{2\,\alpha^2 f^2}\,, \qquad c_1 = \frac{\pi(\mathrm{i}h + f)}{2\,\alpha f^2}\,, \qquad c_2 = \frac{\pi(\mathrm{i}h^2 + \mathrm{i}f^2 + 2hf)}{2f^2}\,; \tag{19}$$

$$\mathcal{W}_{12}: \qquad c_0 = -\frac{\pi}{4\,\alpha^3 f^3}\,, \qquad c_1 = \frac{\pi(\mathrm{i}f - h)}{4\alpha^2 f^3}\,, \qquad c_2 = \frac{\pi(2\,\mathrm{i}hf + f^2 - h^2)}{4\,\alpha f^3}\,; \tag{20}$$

$$\mathcal{W}^2_1: \qquad c_0 = \frac{\pi}{2\,\alpha^3 f^3}\,, \qquad c_1 = \frac{\pi h}{2\,\alpha^2 f^3}\,, \qquad c_2 = \frac{\pi(h^2 + f^2)}{2\,\alpha f^3}\,. \tag{21}$$

Here the singular in α coefficients do not depend on the prescription for poles in τ, since in the considered above examples the nonintegrable terms (for which the prescription is needed) are nonsingular in α.

On the base of the above results let us formulate (and prove) the following properties.

Property 1

The incomplete expansion (17) of the squared propagator $\mathcal{W}(\alpha;\tau)$, considered together with formulas (18)-(21), is equivalent within the given precision to its complete AO expansion.

The validity of this statement may be shown by direct expansion of the corresponding expressions and comparison of the results.

Property 2

Any incomplete expansion of $\mathcal{W}(\alpha;\tau)$ in order to be equivalent within the given precision to its complete AO expansion must involve in denominators all nonzero at $\tau = 0$ contribution to $\mathrm{Im}\Sigma_1(\tau)$. All other contributions to $\Sigma_1(\tau)$ may be transferred from denominators to numerators in the sense of the usual expansion, but by finite number of steps.

The proof we perform by two stages. At first we show that all zero at $\tau = 0$ contributions to $\Sigma_1(\tau)$, without loss of precision, may be transferred from denominators. Then we show that the same operation may be done also for the whole of the real part of $\Sigma_1(\tau)$.

So, let $\Sigma_1(\tau) = \Sigma_{01}(\tau) + \widetilde{\Sigma}_1(\tau)$ where by definition $\widetilde{\Sigma}_1(0) = 0$, but $\Sigma_{01}(0) \neq 0$. As long as in some neighborhood of $\tau = 0$ the function $\widetilde{\Sigma}_1(\tau)$ is a correction one with respect to $\Sigma_{01}(\tau)$, its contribution may be transferred from denominators to numerators, like it was done in (16) and (17) for the higher-order loops. As a result formula (17) becomes as follows:

$$
\begin{aligned}
\mathcal{W}(\alpha;\tau) = \mathcal{W}_1(\alpha;\tau) + &\left[\begin{array}{l} \left(\alpha\,\widetilde{\Sigma}_1 + \alpha^2\Sigma_2 + \alpha^3\Sigma_3\right)\, \mathcal{W}_{11}(\alpha;\tau) \\ + \left(\alpha^2\widetilde{\Sigma}_1^2 + 2\alpha^3\widetilde{\Sigma}_1\Sigma_2 + \alpha^4\Sigma_2^2\right)\, \mathcal{W}_{12}(\alpha;\tau) + \text{h.c.} \end{array} \right] \\
&+ \left[\alpha^2\left|\widetilde{\Sigma}_1\right|^2 + \alpha^3\left(\widetilde{\Sigma}_1\,\overset{*}{\Sigma}_2 + \text{h.c.}\right) + \alpha^4\left|\Sigma_2\right|^2\right] \mathcal{W}_1^2(\alpha;\tau) + O(\alpha^2)\,.
\end{aligned}
\tag{22}
$$

Here symbol $*$ means complex conjugation, and in the denominators of $\mathcal{W}_{1n}^m$ only $\Sigma_{01}(\tau)$ is Dyson resummed. The remainder in (22) is estimated in the AO sense. AO expansions of $\mathcal{W}_1$, $\mathcal{W}_{11}$, $\mathcal{W}_{12}$ and $\mathcal{W}_1^2$ within the required precision are described above in this section.

The proof of formula (22) may be done noticing that within the given precision the quantity $\alpha\widetilde{\Sigma}_1^2$ gives nonzero contribution being exponentiated not more than to quadrate. Really, on background of the regular terms this result is obvious. If $\widetilde{\Sigma}_1^2$ (or $|\widetilde{\Sigma}_1|^2$) is multiplied by a counterterm, then the result may be nonzero in only case of the second and higher derivative of the delta-function (otherwise there acts property $\widetilde{\Sigma}_1(0) = 0$). In functions $\mathcal{W}_{1n}^m = [\mathcal{W}_1]^m\,\Delta_1^n$ ($n \geq 0$, $m \geq 0$, $n+m-1$ is the number of self-energy insertions in one hand of the cut in the diagram of unitarity, $m-1$ does in the another hand) such counterterms appear in order $\alpha^{-(n+2m-1)} \times \alpha^2$, and those only functions $\mathcal{W}_{1n}^m$ can be multiplied by factor $\widetilde{\Sigma}_1^2$ (or $|\widetilde{\Sigma}_1|^2$) which satisfy condition $n+2m-2 \geq 2$. As long as in two insertions of $\widetilde{\Sigma}_1$ each insertion gives a factor α, and in the remaining $n+2m-4$ insertions of $\widetilde{\Sigma}_k$, $k \geq 2$, each insertion gives a factor not less than α^2, one may estimate all mentioned above contributions as $O(\alpha)$. While extending the above reasoning to the third and the higher powers of $\alpha\widetilde{\Sigma}_1$, one may easily see that they give nonzero contributions in the order only $O(\alpha^2)$.

The above result may be generalized to the real part of $\widetilde{\Sigma}_1(\tau)$. In this case $\widetilde{\Sigma}_1(\tau)$ in formula (22) must be defined by $\mathrm{Im}\widetilde{\Sigma}_1(0) = 0$, with $\mathrm{Im}\Sigma_{01}(0) \neq 0$. The basis for this generalization is the observation that the real part of the self-energy does not contribute to the leading term of AO expansion of $\mathcal{W}(\alpha;\tau)$. Let us remark, that formula (22) with this modification on the first sight should look much more complicated because from a formal point of view it should contain infinite series of terms of the type $[\mathrm{Re}\Sigma_1(\tau)]^n\mathcal{W}_{1n}$. However by forming groups with other functions $\mathcal{W}_{1n}^m$ all superfluous terms must be mutually cancelled. The mentioned groups will be formed by virtue of relations of the type $2\mathcal{W}_{12} + \mathcal{W}_1^2 = O(\alpha^{-2})$ [not $O(\alpha^{-3})$], etc. The validity of formula (22) may be verified by direct expanding and comparing the results.

Property 3

There is the following approximation of $\mathcal{W}(\alpha;\tau)$ up to $O(\alpha^n)$ corrections:

$$
\mathcal{W}(\alpha;\tau) = \mathcal{W}_{[n]}(\alpha;\tau) - \alpha^{n-1}\frac{\mathrm{Im}\Sigma_{n+1}(0)}{[\mathrm{Im}\Sigma_1(0)]^2}\,\pi\delta(\tau) + O(\alpha^n). \tag{23}
$$

Here $\mathcal{W}_{[n]}(\alpha;\tau)$ stands for the squared propagator with Dyson resummed self-energy up to n-loops ($n \geq 1$). The second term collects the $(n+1)$-loop correction which has to be added (with the factors) in order to obtain the approximation of $\mathcal{W}(\alpha;\tau)$ up to $O(\alpha^n)$ corrections.

It is worth noticing that within the given precision any admissible, in the sense of Property 2, scheme of incomplete expansion of $\mathcal{W}_{[n]}(\alpha;\tau)$ is practicable, as well as the complete AO expansion. Formula (23) follows immediately from the obvious generalization of formula (17) to the case of any n with taking into consideration the first result in (19).

Formula (23) represents exact quantitative characteristic of the indicated in Introduction property of the reduction of one-loop PT order in the resonance region. Indeed, as long as $\mathcal{W}_{[n]}(\alpha;\tau) \sim \alpha^{-1}$ at $\alpha \to 0$, the second term in formula (23), which involves the $(n+1)$-loop contribution, describes the n-order correction, but not the $(n+1)$-th one as it might be naively expected for $(n+1)$-loop corrections. Since it is impossible to obtain the second term in formula (23) analyzing the amplitude only, it is pertinent to call it *anomalous additive term*.

Property 4

The obtained AO expansions transform as follows when the argument τ shifts on a quantity of order $O(\alpha)$:

$$\widetilde{\mathcal{W}}(\alpha;\tau) = \widetilde{\mathcal{W}}(\alpha;\tau-\alpha\, m^2)\Big|_{\substack{h(\tau-\alpha\, m^2)\to h(\tau)-m^2 \\ f(\tau-\alpha\, m^2)\to f(\tau)}} . \tag{24}$$

Here $\widetilde{\mathcal{W}}(\alpha;\tau)$ stands for AO expansion of any considered above function $\mathcal{W}^m_{1n}(\alpha;\tau)$ or for the initial function $\mathcal{W}(\alpha;\tau)$, quantity m^2 is of order $O(1)$.

Property (24) means non-sensitivity of the formalism with respect to variation of the mass shell within $O(\alpha)$, and also independence of the formalism from UV renormalization scheme. Really, at the one-loop level the transition, for example, from the MS scheme to the on-mass-shell (OMS) scheme is described by

$$\begin{gathered}\Sigma_{OMS}(p^2) = \Sigma(p^2) - \mathrm{Re}\Sigma(M^2_{OMS}) - (p^2 - M^2_{OMS}) \times \mathrm{Re}\Sigma'(M^2_{OMS})\,, \\ M^2_{OMS} = M^2 - \mathrm{Re}\Sigma(M^2_{OMS})\,, \qquad Z_{OMS} = 1 + \mathrm{Re}\Sigma'(M^2_{OMS})\,.\end{gathered} \tag{25}$$

Transformation (25), obviously, belongs to the class of transformations enveloped by formula (24).[6] Transformation at the multi-loop level is inspected by formula (17). Therefore, it can be realized according to the standard recipes of UV renormalization which do not depend on presence of the "infra-red" counterterms, located strictly on the mass shell (see also [15] and the references therein). In addition let us note, that transformation to another scheme of UV renormalization can be carried out (speculatively) for the full Green functions before squaring the amplitude and the AO expansion. The consequent AO expansion by no means "feels" in what scheme the Green functions were determined.

The property (24) is trivial for non-expanded $\widetilde{\mathcal{W}}$ and for their formal expansions (in the sense of usual functions). Nontrivial aspect is that this property remains valid also for counterterm $E(\tau)$. However this property also can be understood if one notes that transformation $\tau \to \tau - \alpha m^2$ does not affect the structure of the homogenization at the scaling $\tau \to \xi\tau$, $\alpha \to \xi\alpha$ (see Section 3). As a result, property (24) appears valid in the most general case.

Below we present one more property which, although do not have connection with the problem of gauge cancellations, represents doubtless independent interest.

[6]Remember, we do not consider contributions to the numerators of propagators. So, we disregard the multiplicative wave function renormalization.

Property 5

From formula (24) there follows the recurrent formula for coefficients c_n,

$$c_{n-1} = \frac{1}{n}\left[\frac{1}{\alpha}\frac{\partial}{\partial h_0} - \sum_{r=0}^{N-n}\left(h_0^{(r+1)}\frac{\partial}{\partial h_0^{(r)}} + f_0^{(r+1)}\frac{\partial}{\partial f_0^{(r)}}\right) + \frac{\partial}{\partial M^2}\right] c_n. \tag{26}$$

Here coefficients $c_n = c_n\left(M^2;\alpha;h_0,\dots h_0^{(N-n)};f_0,\dots f_0^{(N-n)}\right)$ are understood as not depending on τ constants, index n runs values $0 \le n \le N$, with N is the maximum degree of derivative of the delta-function in counterterm $E(\tau)$. Formula (26) is written down with allowance for probable dependence on the parameter M^2 in coefficients c_n. In the considered above examples there are not such dependence, but it arises in configurations in which the interchangings by massless particles are explicitly taken into account [13]. An essential point for derivation of formula (26) is the expansion in powers of α of the delta-function $\delta(\tau-\alpha m^2)$ and of its derivatives in r.h.s. of relation (24). The practical value of formula (26) is that it allows one to determine the "lower" coefficient to within $O(\alpha^L)$ if the "higher" coefficient is known to within $O(\alpha^{L+1})$.

5 Interchangings by massless particles

The problem of taking into consideration massless particles (photons) requires a special analysis because their contributions to self-energy of unstable particles involve a singularity of the type $\tau \times \ln(\tau - \mathrm{i}0)$. The first derivative of this expression is not defined at zero. Therefore, already the first correction term in formula (14) becomes uncertain.

One way to solve this problem is to introduce the regularization mass for massless particles (soft-photon mass). Then, the non-analyticity at $\tau = 0$ disappears in self-energy, and it opens a way to use without problems the obtained above formulas. After the calculation of probability, with taking into account the radiation of the real soft photons and the characteristic cut on their energies, the singular in the soft-mass contributions will be canceled, similarly how it takes place in QED. This property means a continuity of probability as a function of the photon mass. So, while solving qualitative problems one may not worry about its presence since in the final results the dependence on the soft-mass may be eliminated by usual passage to the limit of its zero value.

Another way [13] is based on use of the regularization properties of the parameter α. This method is naturally embedded into the context of the AO expansion problem and permits to automatically ensure cancellation of IR divergences prior to any other calculations. It should be emphasized, that we mean here those IR divergences which origin is connected with emergence of a singularity in α at $\alpha \to 0$. In reality such divergences appear in those diagram configurations in which the soft momenta of massless particles come into the unstable particles lines considered near the mass shell. Let us note, that the cancellation of these IR divergences means cancellation of the corresponding singularities in the coupling constant, and vice versa. The essence of the method proposed in [13] consists in stepwise expansion of the full squared Green functions: first in contributions of the massless particles only, and then in other vertices using the AO technique if it is necessary. Let us note, that the first step in this method is always possible to do due to the Property 2 observed on in this paper and the property $\mathrm{Im}\widetilde{\Sigma}_1(0) = 0$ for the massless particles contributions.

Doing in such a manner, one gets at the intermediate stage the modified Green functions which unstable propagators do not contain the contributions of the massless particles. The payment for this simplification is emergence of definite number of configurations for which some special counterterms are needed. The algorithm allowing enumerate such configurations is described in [13], and there, on the base of unitarity reasons, the general proof of cancellation of the considered class of IR divergences is given. This fact means that the corresponding singularities in the parameter α, arising due to IR divergences, are cancelled. Consequently, the corresponding soft-mass singularities in the alternate scheme, based on the soft-mass regularization, must be canceled, too.

6 Unstable propagators and gauge cancellations in electroweak theory

Now let us show that the modified PT guarantee both the gauge cancellations and the necessary precision of the description in the sense of expansion in powers of the coupling constant. The argumentation we perform in rather general format applied basically to any unstable particle in electroweak theory. The basic idea is to separate in probability, with help of formula (23), the contributions certainly possessing the property of gauge cancellations from the problem contributions. This allow us to concentrate then on study the problem contributions only.

Let us begin with some preliminary notes. First of all let us determine the photon-mass regularization for IR divergences. Since IR divergences are cancelled in probability, the probability is a continuous function of the soft-photon mass. Therefore the dependence on it may be eliminated in the final results by usual passage to the limit. So, if one proves the property of gauge cancellations in the presence of soft-photon mass, then after the passage to the limit of its zero value the result must take place, as well.[7]

The photon mass insertion allows one to solve at once the following two problems. First, owing to accompanying correction of the analytical properties of self-energy there appears an opportunity of direct application of formula (23) (see discussions in Sections 3 and 4). Second, the diagram configurations with emittion/absorption of the soft photons are managed to be referred completely to the vertex blocks. As a result, the problem of instability is reduced solely to configurations with directly outgoing particles from the interaction region.

Now let us discuss the unphysical pole contributions to the vector boson propagators. Assuming the next parameterization for self-energy

$$\Sigma_{\mu\nu}(p) = \Sigma\, g_{\mu\nu} + \Sigma_L\, p_\mu p_\nu, \tag{27}$$

we obtain the following explicit expression for the full propagator in R_ξ-gauge:

$$D_{\mu\nu}(p) = \frac{g_{\mu\nu} - p_\mu p_\nu/p^2}{p^2 - M^2 + \Sigma} + \xi\, \frac{p_\mu p_\nu/p^2}{p^2 - \xi\, M^2 + \xi\, (\Sigma + p^2\Sigma_L)}. \tag{28}$$

[7]The photon mass may be inserted without violation of WI responsible for $U(1)$ invariance. One can see this considering the problem in Stueckelberg formalism (the author is grateful to A.A.Slavnov for indication on this fact). Besides, the photon mass may be inserted by the totally gauge invariant fashion [21].

Here the first term represents the product of the spin factor on unstable propagator $\Delta(\alpha;\tau)$ introduced by formula (1). The second term describes the unphysical pole contribution. Let us remind that in the amplitude, considered in framework of the conventional PT, the contributions of the second term due to WI are cancelled by contributions of other unphysical states. So, the second term in (28) does not lead to nonintegrable singularity in the phase space. This fact allows us in the modified PT to take into account the second term in (28) in the conventionally expanded form, as well. Then, the property of gauge cancellations of its contributions will be checked by properties of the first term in (28), taken by modulus and squared. It should be noted, that the cross terms resulting from the operation of squaring propagator (28) do not lead to nonintegrable singularities in the phase space. Therefore, they also may be taken in the conventionally expanded form.

Now let us show that contributions of the first term in propagator (28), being taken into consideration in framework of the modified PT, does not break gauge cancellations. The proof we begin with analysis of contributions of the anomalous (second) term in formula (23). Due to its additivity the simplest way to do such analysis is to assume (temporarily) that the squared unstable propagator consists in this anomalous term only.

At first we consider the case of single unstable particle production. Note, that the complementary part of the diagram of unitarity with respect to the given squared propagator, by virtue of the delta-function *without a derivative* in the anomalous term, is taken strictly on the mass shell. From this fact it immediately follows that the mentioned complementary part of the diagram of unitarity is gauge invariant, because actually it coincides (up to factors) with the product of two squared S-matrix elements, that describe the on-shell production and decay of the unstable particle. Let us emphasize, that the details of definition of the mass shell within the $O(\alpha)$ corrections are inessential, because the second term in formula (23) describes the highest-order contribution within the given precision.

In the case of multiple production of unstable particles the contributions from all other unstable particles — except the given one — should be considered, for the same reason, in the leading order of AO expansion. By virtue of (6), (8), and (14) the leading order contributions are determined by the delta-function, again *without a derivative*, with the coefficient involving the one-loop f_0 only as a nontrivial factor. In view of the gauge invariance of the one-loop f_0 (which is seen, for instance, from its explicit expression [5]) from here there follows, once again, the gauge invariance of the complementary part of the diagram of unitarity with respect to the given squared propagator. Due to the additivity, the similar reasoning may be repeated for other unstable squared propagators, and in each case from the gauge invariance of the corresponding complementary parts of the diagram of unitarity there will follow the property of gauge cancellations in the relevant contributions to probability.

Now let us proceed to analysis of contributions of the first term in formula (23) and, simultaneously, to analysis of all other accompanied contributions to probability (including the non-resonant contributions). The proof of gauge cancellations among these contributions may be easily done in framework of the background-field formalism. One should only take into advantage the fact that in the background-field formalism the Dyson resummation of self-energy up to n-loops do not violate WI provided that in the vertex functions all contributions up to n-loops are taken into account, as well [10]. Treating the quantity $\mathcal{W}_{[n]}(\alpha;\tau)$ in formula (23) in this way, one automatically gets the property of gauge cancellations in probability. From here it immediately follows that in the complete AO expansion the gauge cancellations within the considered precision take place, too. The above reasoning complete the proof of gauge cancellations in the modified PT.

Two important notes should be done in the conclusion of this section. First, in the above reasoning the usage of any incomplete expansion of $\mathcal{W}_{[n]}(\alpha;\tau)$ is, generally speaking, inadmissible (an exception see in the next section). The matter is that any incomplete expansion *a priori* does not guarantee the property of exact gauge cancellations, since such cancellations must take place only within the given precision of the expansion, but not beyond this precision. However, any incomplete expansion always contains superfluous contributions which are not under control. Another matter is the complete (exact) AO expansion in which all superfluous contributions are cut by definition.

The second note is that the above result may be obtained, in principle, by starting with the analysis of directly $\mathcal{W}_{[n+1]}(\alpha;\tau)$. However for this purpose one should know beforehand with what precision this quantity describes the squared propagator $\mathcal{W}(\alpha;\tau)$. The study of the present paper gives an exact answer to this question.

7 Generalization of the fermion-loop scheme

In case when the analysis can be restricted by NLO precision, as, for example, in the case of W-pair production on LEP2, the gauge cancellations may be proved in the usual formalism, without the treatment for the background-field method. The key point is the well-known result on gauge cancellations in the so-called fermion-loop scheme [8, 9]. Remind, it consists in including all fermionic one-loop corrections in tree-level amplitudes and Dyson resumming the self-energies. The difficulties of this scheme are the vagueness in including also the bosonic corrections without spoiling the gauge cancellations, and the problem of taking into account the two-loop corrections to self-energy in denominators of unstable propagators, which as we know (see also [11, 12]) are necessary for completing the NLO approximation.

Both these problems may be solved within the modified PT approach with usage of the AO technique. Let us begin with the two-loop corrections to self-energy. As we have seen, they can be simply taken into consideration by adding the anomalous term to the probability, calculated in the usual approach with Dyson resummed one-loop self-energy corrections. In view of (23) this operation may be carried out by means of the formula

$$\alpha\, \mathcal{W}(\alpha;\tau) = \alpha\, \mathcal{W}_{[1]}(\alpha;\tau) - \alpha\, \frac{\mathrm{Im}\Sigma_2(0)}{\left[\mathrm{Im}\Sigma_1(0)\right]^2}\, \pi\delta(\tau) + O(\alpha^2). \tag{29}$$

Here $\mathcal{W}_{[1]}(\alpha;\tau)$ represents the squared unstable propagator in which denominator all one-loop corrections are Dyson resummed. The additional factor α is inserted in (29) in order to designate the decay block of unstable particle. Remember, $\alpha \times \mathcal{W}_{[1]}(\alpha;\tau) = O(1)$ at $\alpha \to 0$.

Let us turn now to the problem of one-loop bosonic corrections. We group them into two classes. To the first class we refer the corrections to self-energy of unstable particles. To the second class we refer the corrections to the vertex factors, and also the corrections caused by the real photons (remind, the photons are considered with nonzero mass till the very end of calculations). The corrections from the first class can be easily taken into account due to the zero-value of the imaginary parts of on-shell bosonic corrections to W- and Z-boson self-energy [5]. Owing to this property and formula (22) they can be, without loss of precision, transferred out from the denominators of unstable propagators. Moreover, in an OMS-like UV renormalization scheme, with the renormalized self-energies are satisfied the condition $\mathrm{Re}\Sigma_1(0) = \mathrm{Re}\Sigma_1'(0) = 0$, there is the following relation:

$$\alpha\, \mathcal{W}_{[1]}(\alpha;\tau) = \alpha\, \mathcal{W}_{1F}(\alpha;\tau) + O(\alpha^2). \qquad (30)$$

Here $\mathcal{W}_{1F}(\alpha;\tau)$ represents the squared propagator with the Dyson resummed only the fermionic one-loop corrections. Substituting (30) into (29) we obtain the formula, on which base we may easily obtain the result on gauge cancellations. To this aim we should only repeat the reasonings of the previous section using the well-known result of gauge cancellations in the fermion-loop scheme (with taking into account the fermionic one-loop corrections to the vertex functions).

However, the bosonic corrections to the vertex functions and the real photon contributions have yet not been taken into consideration. In order to do this let us make use of the fact that in the presence of the bosonic corrections, but within the NLO precision, the quantity $\mathcal{W}_{1F}(\alpha;\tau)$ must be taken in the leading order approximation only. Remember, $\mathcal{W}_{1F}(\alpha;\tau) = \pi/(\alpha f_{0F}) \times \delta(\tau) + O(1)$. Here the leading term is explicitly gauge invariant and due to the delta-function *without a derivative* all factors that appear in the diagram of unitarity are also gauge invariant (see the previous section).

So, summarizing the results we come to the following recipe of the generalization. We formulate it having in mind the total cross section for the typical LEP2 processes CC10, CC11 and CC20, which have been studied in framework of the fermion-loop scheme in [9]. The starting point of the generalization, of course, is the probability obtained in the fermion-loop scheme proper, i.e. without any bosonic corrections. At this stage the gauge cancellations are guaranteed [9]. However the precision of the description — in the sense of expansion in powers of the coupling constant — so far is insufficient, since still there are other corrections within the NLO approximation.

The mentioned corrections may be collected in the two following terms. The first term describes the anomalous contributions. Its structure follows the fact that among all anomalous-term contributions to the probability in NLO approximation there survive only those contributions which correspond to the pair on-shell production of unstable particles. Really, let us consider all the off-shell subprocesses. Among them those only contribute in the leading AO order which are mediated by the pair vector boson production, since these only subprocesses include the factor $1/\alpha^2$ originating from the product of two squared unstable propagators. (In fact these subprocesses belong to CC03 class.) The presence of the extra factor α in the anomalous term transfers their contribution to the correction-type ones, i.e. to NLO.

The second term represents the same probability of the pair on-shell production, but now with the bosonic corrections (excluding, of course, the correction to the self-energy of unstable particles which has already been taken into account by the use of formula (30)). An example of corrections of this type are the diagrams that contribute to the Coulomb singularity, because with nonzero photon mass the corresponding configurations may be referred to the vertex factors. The corrections to the cross section, caused by the real soft photon radiation/absorption by an unstable particle, must also be included into the second term, but only if its partner in the interchanging process does not belong to the decay block of another unstable particle. All other so-called "non-factorizable" corrections [11, 12] should not be taken into account, since with nonzero photon mass they split the momenta at least in one pair of the mutually conjugated unstable propagators in both sides of the diagram of unitarity. As a result, the corresponding pair of propagators loses ability to be of the modulus-squared type and, correspondingly, loses the ability to generate the leading contribution of the type $1/\alpha$. The presence of the extra factor α, specified by the one-photon interchanging (independently whether the photon real or virtual), leads the given contribution beyond the NLO approximation. Let us remark, that this strict result agrees with observation [11, 12] about suppression of the non-factorizable corrections with respect to the factorizable ones, obtained in the pole scheme in double-pole approximation.

The above discussion leads us to the next formulas for the total cross section:

$$\sigma(s) = \int\limits_0^s ds_+ \int\limits_0^{(\sqrt{s}-\sqrt{s_+})^2} ds_- \; \sigma_0(s\,;s_+,s_-), \tag{31}$$

$$\begin{aligned}\sigma_0(s\,;s_+,s_-) = \sigma_0^{\text{off-shell, fermion-loop-scheme}}(s\,;s_+,s_-)\\ +2\,\sigma_0^{\text{on-shell, tree}}(s\,;M_+,M_-)\times\prod_{\kappa=\pm}\delta(s_\kappa-M_\kappa^2)\times\alpha\,\mathrm{Im}\Sigma_2(0)/\mathrm{Im}\Sigma_1(0)\times\mathrm{BR}_\kappa\\ +\sigma_0^{\text{on-shell, boson-one-loop + real-photon}}(s\,;M_+,M_-)\times\prod_{\kappa=\pm}\delta(s_\kappa-M_\kappa^2)\times\mathrm{BR}_\kappa\,.\end{aligned} \tag{32}$$

Here BR_κ means the branching of unstable particle evaluated on-shell. Factor 2 in the second term in (32) corresponds to the presence of two intermediate unstable particles. In formula (32) we have used the relation $\mathrm{Im}\Sigma_1(0) = M\Gamma_0(M)$ following from unitarity, with $\alpha\Gamma_0(M)$ is the on-shell width calculated in the tree approximation (one can verify this relation by direct calculations [22]).[8] It should be noted, that $\sigma_0(s\,;s_+,s_-)$ is not an observable quantity. So it is not surprising that it involves the delta-functions (see also the next section).

In formula (32) one may do further simplification by proceeding to the complete AO expansion. For this purpose one can use the next formula which is valid in the OMS-like scheme of UV renormalization:

$$\alpha\,\mathcal{W}_1(\alpha;\tau) = [\mathrm{Im}\Sigma_1(0)]^{-1}\;\pi\delta(\tau) + VP\frac{\alpha}{\tau^2} + O(\alpha^2). \tag{33}$$

Substituting (33) into (32) one finally obtains

$$\begin{aligned}&\sigma_0(s\,;s_+,s_-) =\\ &\quad VP\sigma_0^{\text{on/off-shell, tree}}(s\,;M_+,s_-)\times\delta(s_+-M_+^2)\times\mathrm{BR}_+\\ &\quad +VP\sigma_0^{\text{off/on-shell, tree}}(s\,;s_+,M_-)\times\delta(s_--M_-^2)\times\mathrm{BR}_-\\ &\quad +2\,\sigma_0^{\text{on-shell,tree}}(s\,;M_+,M_-)\times\prod_{\kappa=\pm}\delta(s_\kappa-M_\kappa^2)\times\alpha\,\mathrm{Im}\Sigma_2(0)/\mathrm{Im}\Sigma_1(0)\times\mathrm{BR}_\kappa\\ &\quad +\sigma_0^{\text{on-shell, tree + one-loop + real-photon}}(s\,;M_+,M_-)\times\prod_{\kappa=\pm}\delta(s_\kappa-M_\kappa^2)\times\mathrm{BR}_\kappa\,.\end{aligned} \tag{34}$$

[8] In the two-loop approximation the similar relation between Σ and Γ most likely does not exist. Anyway it does not follow from unitarity.

Here the first two terms, as well as obvious contributions to the last term, originate from the first term in formula (32). Namely, the first two terms in (34) accumulate the contributions of the mixed nature originating from subprocesses of the pair vector boson production (CC03) and simultaneously from subprocesses of the single vector boson production (the other subprocesses of CC10, CC11 or CC20). In the first case (CC03) the symbol VP means that in accordance with (33) the corresponding off-shell unstable squared propagator is approximated by $VP\,1/\tau^2$, while another unstable particle is considered as produced/decayed on-shell. In the case of the single vector boson production in the first two terms the symbol VP is superfluous and may be omitted. It should be noted, that the usage of prescription of the principal value in the first case is necessary, because the usage of another prescription for $1/\tau^2$ in (30) and (33) may lead to additional contributions in r.h.s. in formula (34). Let us emphasize, that this remark does not concern the anomalous term which is described by the third addend in formula (34), since the anomalous term arises from the singular in α contributions to the function $\mathcal{W}_{11}(\alpha;\tau)$ (see formula (19) and the note after formula (21)).

The obtained above results can be easily extended to any other process with unstable particles production, including other processes of 'CC'-type and also processes of 'NC'-type. In general, it is clear how to write down the similar formulas for any process mediated by unstable particles.

8 Discussion

In this paper we have proved the property of gauge cancellations in probabilities of processes, determined in framework of the modified PT with usage of the AO technique. In any order of the expansion in the coupling constant we have proved this property by taking into advantage of the background-field formalism. Within the one-loop precision (NLO) we have found the proof in the usual formalism, as well, by applying the results of the fermion-loop scheme. However, in contrast to the pure fermion-loop scheme, we have taken into consideration all corrections that are necessary in NLO approximation. From the practical point of view the latter result, apparently, is the main one obtained in the present paper.

It should be noticed, that the result on gauge cancellations was expected since appearance of Ref.[13], where it was shown that calculation of probability of a processes mediated by unstable particles may be reduced to the regular fixed-order calculation. The latter fact gave a reason to think that the problem of gauge invariance was practically solved, as well. However, on closer examination of the problem it has been revealed that there is a large distance before deriving the strict result. Indeed, the Feynman rules in the modified PT + AO coincide with the usual ones only outside the point of the on-mass-shell singularity in unstable propagator. Therefore the gauge invariance *a priori* takes place only in this area of the kinematic variables. But in any neighborhood of the mentioned singularity the nonstandard contributions, resulted due to the delta-functions and VP prescriptions, are *finite* independently of how the neighborhood is small. Moreover, *a priori* it is not known anything of the properties of these contributions. This is especially clear in the higher orders of the AO expansion (beginning with NLO) where the delta-functions contribute with derivatives. This fact suppress direct application of the statement about the gauge invariance of the complementary configurations in the diagrams of unitarity with respect to the given squared propagator (see Section 6).

In the present paper the mentioned difficulty is avoided by attracting some specific properties of the AO expansion of the squared unstable propagator, permitting stepwise comparison with the results of the usual approach based on the calculation the amplitude with Dyson resummation. The special role in this scheme is assigned to the anomalous additive term, which corrects the results obtained within the usual approach.

In general, the method of a comparison with results of the usual approach promotes to derive results in a shorter way. However, on the other hand, it not always allows one to reveal the full advantages of the modified PT + AO. In particular, being applied to the higher orders of the modified PT with the usage of the background-field formalism, this method does not allow one to solve *completely* the problem of gauge invariance. It is shown only the gauge cancellations that are inspected by WI. This is enough to avoid the uncontrolled high-energy contributions and large contributions associated with the ratios of the kind s/m_e^2 etc. [8]. Nevertheless, some residual dependence on the quantum gauge parameter could remain, as long as it remains after the gauge cancellations in results of Ref.[10]. In Refs.[9, 12] this property was interpreted as an indication on "arbitrariness" to some extent of any resummation.

However, while taking into account the anomalous additive term and carrying out the complete AO expansion one can expect that such residual dependence would disappear. Really, let us look at formula (32), imagining that it is obtained in the background-field formalism. In its first term in r.h.s. there can remain some residual dependence on the quantum gauge parameter [10]. But it can emerge also in the second term through $\mathrm{Im}\Sigma_2(0)$. In contrast to its one-loop analog, the quantity $\mathrm{Im}\Sigma_2(0)$ is not connected with the on-shell width. Consequently, in the background-field formalism there is not injunction for $\mathrm{Im}\Sigma_2(0)$ to be necessarily gauge invariant. Therefore, proceeding from formula (32) to formula (34) one may quite well expect the cancellation of the mentioned dependence on the quantum gauge parameter.

Now let us briefly discuss the result which is probably the most important, at lest from the practical point of view. We mean the elementary generalization of the fermion-loop scheme which possesses both the gauge cancellations and the necessary precision of the description in NLO approximation. It has appeared that the anomalous term is the only up to now not calculated ingredient of this generalization. Actually, one may consider the proposed generalization as an instruction how to use the already known results of the former calculations. In this connection we call attention to the result about the absence in NLO approximation of the so-called non-factorizable corrections in processes of W-pair production. (Nevertheless, one still needs to consider the configurations describing the Coulomb singularity and some configurations with real photon emittion/absorption directly by unstable particle. See Section 7 for the detailed description. In addition, note that it is necessary also the convolution by the purely initial-state and purely final-state flux functions.) This means that within NLO precision there is not need to attract the special technique of calculation of additional counterterms caused by the soft-photon interchanging, which was designed in Ref.[13] (see the discussion in Section 5).

In conclusion, let us discuss two more problems which usually arise at comparison of the results of the modified PT + AO with the results of the usual approach. The topic, as a matter of fact, is a comparison with the well-known Breit-Wigner parameterization of contributions of unstable states. The first problem concerns the definition of the "physical" mass and width of unstable particle. In the modified PT + AO both these quantities are secondary ones (or pseudo-observables [5, 22]), which have to be defined on the base of the primary objects, such as the renormalized (non-physical) Lagrangian mass, coupling constant, etc. Apparently, the most radical way consists in the identification of these quantities with the position of the pole in the complex plane in the full unstable propagator, which is equivalent to finding solution to the equation $M^2 - s_p - \Sigma(s_p) = 0$, with $s_p = M_p^2 - \mathrm{i} M_p \Gamma_p$. Notice, the latter operation may be done perturbatively.

The second problem concerns the presence of the delta-functions in the results of the AO expansion, and, most likely, it is the most actual problem from the point of view of adaptation of the modified PT + AO method in scientific community. The problem may be designated as an illusory discrepancy between the presence of the delta-functions and the notions about a continuity of the physical results as functions of the physical parameters (kinematic variables). This problem, of course, has been stated and discussed in Ref.[13]. The essence of the solution is reduced to the observation that there is not actually direct identity between the squared amplitude (that is usually calculated) and the genuine probability of the process (that is observed). Really, a formal expression for probability, which is calculated on the base of diagrams of unitarity, before it becomes an observable quantity, should necessarily be subjected to operation of an *integration*, or "*smearing*" with some weight function.[9] For instance, in the case of the differential cross section, in fact, one needs only the probability integrated over the corresponding bin of kinematic variables. Indeed, in experiment only an integrated over the bin quantity can be observed. So, the task of the theory is to calculate explicitly this quantity.

It is worth noticing that the necessity of using the binning trick is not a result of imperfection of only the given experimental device. The matter in that even using technically perfect equipment the effect of binning all the same will remain, since the measuring devices by principal reasons always influence on the measuring object [23]. In the context of the considered problem this means that in the Nature there may not be devices that are capable to generate absolutely monochromatic beams and simultaneously to register an absolutely monochromatic final states. Such devices do not exist and can not exist because in both cases the appropriate (ideal) devices should be [24] by the size of the Universe.

Thus, during any measuring procedure there is not even a hypothetical expedient to "hit" precisely into the delta-function, because any measuring at once necessarily envelops a neighborhood. The property of smoothness of the calculated results in case of the hit into the neighborhood which includes contributions of the delta-functions is controlled by the asymptotic properties of the AO expansion.

Acknowledgements

The author is grateful to D.Yu.Bardin, L.V.Kalinovskaya, and F.V.Tkachov for discussions of various problems connected with applications of the electroweak theory. Special thanks to F.V.Tkachov for invitation to study the problems of instability and consultation about the results of his work [13]. The author would also like to thank A.A.Slavnov for

[9] It is necessary to remind, that in case of absence of an integration there is not the problem of nonintegrable singularities in the phase space. So, all calculations can be carried out in the usual PT.

the valuable note about WI in presence of the photon mass, and E.E.Boos for indication on importance of the single-resonant subprocesses in W-pair production. It is a pleasure to thank B.A.Arbuzov, V.E.Rochev and S.R.Slabospitsky for support and useful discussions.

This work was supported in part by the Russian Foundation for Basic Research, grant No. 99-02-18365.

References

[1] Report CERN 96-01, Physics at LEP2, eds. G.Altarelli, T.Sjostrand and F.Zwirner.

[2] M.Veltman, Physica 29 (1963) 186.

[3] W.Beenakker et al., in Ref. [1], p.79.

[4] G.Montagna, O.Nicrosini and F.Piccinini, Rev.Nouvo Cim. 21 (1998) 1.

[5] D.Bardin and G.Passarino, The Standard Model in the Making Precision Study of the Electroweak Interactions, Oxford Science Pub., Clarendon Press, Oxford,1999.

[6] R.G.Stuart, Phys.Lett. B262 (1991) 113.

[7] W.Beenakker, F.A.Berends and A.P.Chapovsky, Nucl.Phys. B548 (1999) 3; A.Denner, S.Dittmaier, M.Roth and D.Wackeroth, Radiative corrections to $e^+e^- \to WW \to 4$ fermions, hep-ph/9909363.

[8] E.N.Argyres et al., Phys. Lett. B358 (1995) 339.

[9] W.Beenakker et al., Nucl.Phys. B500 (1997) 255.

[10] A.Denner and S.Dittmaier, Phys.Rev. D54 (1996) 4499.

[11] S.Dittmaier, Theoretical aspects of W physics, in Proc. HEP 97 (Jerusalem), eds. D.Lellouch, Springer-Verlag, 1999, p.709; Radiative corrections to W-pair production in e^+e^- annihilation, hep-ph/9811434.

[12] W.Beenakker and A.Denner, Acta Phys.Polon B29 (98) 2821.

[13] F.V.Tkachov, Perturbation theory With unstable fundamental fields, hep-ph/9802307.

[14] F.V.Tkachov, Int.J.Mod.Phys. A8 (1993) 2047.

[15] F.V.Tkachov, Phys.Part.Nucl., 25 (1994) 649.

[16] F.V.Tkachov, Phys.Lett. B412 (1997) 350.

[17] L.Schwartz, Theorie des Distributions. I, II, Paris, 1950-51.

[18] I.M.Gelfand and G.E.Shilov, Generalized Functions. Academic Press, 1968.

[19] N.N.Bogolyubov, Doklady USSR Acad. Sci., 82 (1952) 217.

[20] N.N.Bogolyubov and D.V.Shirkov, Introduction to the theory of quantized fields. Fizmatgiz. Moscow, 1957 (English transl.: Wiley, 1980, 3rd ed.)

[21] A.A.Slavnov, Phys.Lett. B98 (1981) 57.

[22] D.Bardin, Field Theory and Standard Model. – In: 1999 European School of High Energy Physics. CERN Yellow Report, 2000 (to be published).

[23] N.Bohr, Nature, Suppl. 121 (1928) 580; Discussion With Einstein on Epistemological Problems in Atomic Physics, in "A.Einstein, philosopher-scientist", Evanston,1949, p.201-241.

[24] W.Heisenberg, Z.Phys. 43 (1927) 172.